On generalized regular and iregular sampling in shift-invariant spaces

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This work concerns the problem of stable recovering of any function f in a shift-invariant space $V_\varphi := \Big\{ f(t) = \sum_{n\in\mathbb{Z}} a_n \ \varphi(t-n) \ : \ \{a_n\} \in \ell^2(\mathbb{Z}) \Big\} \subset L^2(\mathbb{R})$, from the regular samples $\{(\mathcal{L}_j f)(rn)\}_{n\in\mathbb{Z},j=1,2,\ldots,s}$ or the irregular samples $\{(\mathcal{L}_j f)(rn+\varepsilon_{j,n})\}_{n\in\mathbb{Z},j=1,2,\ldots,s}$ of some filtered versions $\{\mathcal{L}_j f\}_{j=1,2,\ldots,s}$ of the function itself. The sampling period $r \in \mathbb{N}$ necessarily satisfies $r \le s$. Starting from a suitable representation for the samples [1], and having in mind that any shift-invariant space with stable generator φ is the image of $L^2(0,1)$ by means of a bounded invertible operator [3], the generalized regular sampling is derived from some dual frame expansions in $L^2(0,1)$ [1]. As a consequence of the representation of the irregular samples, generalized irregular sampling arises from the theory of perturbation of frames [2]. The generalized irregular sampling results can be expressed in terms of $\delta := \sup_{j,n} |\varepsilon_{j,n}|$, and they can be implemented by means of the frame algorithm. Finally, some examples in the space of cubic splines are provided.

References

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