Augmented Lagrangian methods with general lower-level constraints are considered in the present research. These methods are useful when efficient algorithms exist for solving subproblems where the constraints are only of the lower-level type. Two methods of this class are introduced and analyzed. Inexact resolution of the lower-level constrained subproblems is considered. Global convergence is proved using the Constant Positive Linear Dependence constraint qualification. Conditions for boundedness of the penalty parameters are discussed. The reliability of the approach is tested by means of an exhaustive comparison against Lancelot. All the problems of the Cute collection are used in this comparison. Moreover, the resolution of location problems in which many constraints of the lower-level set are nonlinear is addressed, employing the Spectral Projected Gradient method for solving the subproblems. Problems of this type with more than $3 \times 10^6$ variables and $14 \times 10^6$ constraints are solved in this way, using moderate computer time.