SPATIAL HOMOGENEITY IN AN ATMOSPHERIC PROBLEM

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Abstract

Let $\Omega \subset \mathbb{R}^N$, $\mathbb{N} \ni N \geqslant 2$, be a bounded domain with smooth boundary $\Gamma := \partial\Omega$. Let Ω_i, $1 \leqslant i \leqslant n$, be a subdomains of Ω with smooth boundary Γ_i such that $\Gamma_i \cap \Gamma_j = \emptyset$, $1 \leqslant i < j \leqslant n$ and let $\Omega_0 = \Omega \setminus \cup_{i=1}^{n} \Omega_i$. Consider the following problem

$$
\begin{cases}
\frac{\partial G}{\partial t} &= \frac{K}{\varepsilon}\Delta G + f(G) \qquad t > 0,\ x \in \Omega_0 \\
\frac{K}{\varepsilon}\frac{\partial G}{\partial \vec{n}_i}(t,x) &= A_i - G \qquad t > 0,\ x \in \Gamma_i,\ 1 \leqslant i \leqslant n \\
\frac{K}{\varepsilon}\frac{\partial G}{\partial \vec{n}}(t,x) &= 0, \qquad x \in \Gamma \\
\frac{\partial A_i}{\partial t} &= \frac{K}{\varepsilon}\Delta A_i + g(A_i) \qquad t > 0,\ x \in \Omega_i,\ 1 \leqslant i \leqslant n \\
\frac{K}{\varepsilon}\frac{\partial A_i}{\partial(-\vec{n}_i)}(t,x) &= G - A_i \qquad x \in \Gamma_i,\ 1 \leqslant i \leqslant n,
\end{cases}
\tag{1}
$$

where $\frac{\partial G}{\partial \vec{n}} = \langle \nabla G, \vec{n} \rangle$, $\frac{\partial A_i}{\partial \vec{n}_i} = \langle \nabla A_i, \vec{n}_i \rangle$, $\vec{n}$ is the normal vector to Γ point outward Ω, $\vec{n}_i$ is the normal vector to Γ_i point inward Ω_i, $1 \leqslant i \leqslant n$. The nonlinearities $f, g : \mathbb{R} \to \mathbb{R}$ are locally Lipschitz functions. Assume without loss of generality that $|\Omega| = 1$. The problem (1) arise in atmospheric problems.

Our aim is to show that, for suitably small $\varepsilon > 0$, the asymptotic behavior of (1) is essentially the same as the asymptotic behavior of the following system of ordinary differential equations:

$$
\begin{cases}
\dot{G}(t) &= f(G(t)) + \sum_{i=1}^{n} |\Gamma_i|(A_i - G) \\
\dot{A}_i(t) &= g(A_i(t)) + |\Gamma_i|(G - A_i), \quad 1 \leqslant i \leqslant n.
\end{cases}
\tag{2}
$$