Superlinear Equations with nonlinearities limited by powers

Sebastián Lorca *

Departamento de Matemática–Universidad de Tarapacá

Casilla 7-D, Arica - Chile, slorca@uta.cl

Abstract

In this work we consider the equation

$$\begin{cases} -\Delta_p u &= f(x, u, \nabla u) \quad \text{in } \Omega\,, \\ u &= 0 \qquad\quad \text{on } \partial\Omega\,. \end{cases} \tag{P}$$

defined in a bounded and smooth domain $\Omega \subset \mathbb{R}^N$, with $N > 2$ .Here $-\Delta_p u = div(|\nabla u|^{p-2} \nabla u)$ is the $p-$ Laplacian operator, with $1 < p < N$. We assume that $f : \Omega \times \mathbb{R}^+ \times \mathbb{R}^N \to \mathbb{R}^+$ is a locally Lipschitz function satisfying the following condition:

$$s^q - M\,|\eta|^\alpha \leq f\,(x, s, \eta) \leq M\,(s^r + |\eta|^\alpha) \qquad \text{for all } (x, s, \eta) \in \Omega \times \mathbb{R}^+ \times \mathbb{R}^N, \tag{f_1}$$

where, $M > 0$; $q, r \in (p-1, \frac{N(p-1)+p}{N-p})$; $\alpha \in (p-1, pq/\,(q+1))$.

In 2004, D. Ruiz have obtained a priori estimate for positive solutions of (P) under the assumption $q = r$ (see J. Differential Equation 199). Here we consider a more general case and we use a more simple arguments to obtain a priori estimate and consequently an existence result:

Theorem 0.1 *Given $q \in (p-1, \frac{N(p-1)+p}{N-p})$ there exists $\varepsilon > 0$ such that problem (P) has at least one positive solutions for any function f verifying (f_1) with $r \in [q, q+\varepsilon)$.*

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