RIEMANN SOLUTIONS OF BALANCE SYSTEMS FOR REACTIVE FLOW IN POROUS MEDIA

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Mass interchange between different phases occurs typically in reactive flows in porous media. Such flows are not represented by conservation laws or hyperbolic equations both because they have source terms and because they are not evolutionary in all the variables. In the absence of gravitational effects, such flows can be modelled as system of $m+1$ equations:

$$\frac{\partial}{\partial t}\mathcal{G}(\mathcal{V}) + \frac{\partial}{\partial x}u\mathcal{F}(\mathcal{V}) = q(\mathcal{V}), \tag{0.1}$$

where $\mathcal{V} : \mathbb{R} \times \mathbb{R}^{+} \longrightarrow \Omega \subset \mathbb{R}^{m}$ represents the dependent variables $\mathcal{V}(x,t)$ to be determined associated to $\mathcal{G}$ and $\mathcal{F}$. The real-valued dependent variable $u(x,t)$ is called speed because this is its interpretation in many applications; the pair $(\mathcal{V}, u)$ in $\mathbb{R}^{n+1}$ is called *state variable.* $\mathcal{G}$ and $\mathcal{F}$ represent the vector-valued functions $\mathcal{G} = (\mathcal{G}_1, \mathcal{G}_2, \cdots, \mathcal{G}_{m+1})^T$: $\Omega \longrightarrow \mathbb{R}^{m+1}$ and $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2, \cdots, \mathcal{F}_{m+1})^T$: $\Omega \longrightarrow \mathbb{R}^{m+1}$, where $u\mathcal{F}_i$ is the flux for the conserved quantity $\mathcal{G}_i$ and $\partial\mathcal{G}_i/\partial t$ is the corresponding accumulation term, for $i = 1, 2, \cdots, m+1$. The terms $q = (q_1, q_2, \cdots, q_{m+1})$: $\Omega \longrightarrow \mathbb{R}^{m+1}$ represent the chemical reaction or mass transfer. The functions $\mathcal{G}$ and $\mathcal{F}$ are continuous in the whole domain Ω, but they are only piecewise smooth. The terms q are generically piecewise continuous functions. Eq. (0.1) has another important feature: the variable u does not appear in the accumulation term, but only in the flux term (0.1). Generically, the equation (0.1) models flows where there are chemical reactions or phase changes giving rise to mass exchange between different phases. In any such flow the balance system (0.1) reduces in several particular physical situations to simpler systems of conservation laws with fewer equations of the form:

$$\frac{\partial}{\partial t}G(V) + \frac{\partial}{\partial x}uF(V) = 0. \tag{0.2}$$

In each physical situation, the corresponding set of variables V is a subset of the set of variables $\mathcal{V}$; F and G are obtained from $\mathcal{F}$ and $\mathcal{G}$; they have $n+1$ components, $n \leq m$.

As example, we show a problem of steam injection. This type of injection is widely studied in Petroleum Engineering and has several applications in practical problems. For the Riemann problem, we consider a one-dimensional porous medium filled with steam, where a mixture of water, steam and nitrogen is injected. An application of this technique is the recovery of geothermal energy. Injecting this mixture into a rock saturated with steam with temperature higher than T^b, the water in the mixture evaporates and we can recover rock thermal heat in the gaseous phase.

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