Hierarchical Spherical Model as a Viscosity Limit of $O(N)$ Heisenberg Model

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A continuum version of the Dyson hierarchical approximation to the $O(N)$ classical Heisenberg model is considered in order to understand dynamical properties of the Lee–Yang zeros of spin systems under renormalization group (RG) transformation.

We combine two tools in our analysis. The first is related to the "proximity" to Berlin–Kac spherical model as N goes to infinity. The second is a recursion relation to "a priori" block–spin distribution derived for hierarchical models.

Watanabe [**?**] has recently explored both tools together with the Lee–Yang property of "a priori" measures to establish renormalization group convergence to a Gaussian measure for $d = 4$ dimensions and N large at the critical inverse temperature.

The renormalization group in the continuum reduces to a semilinear parabolic differential equation (PDE)

$$w_t = -\frac{1}{2N} w_{yy} + w w_y - \frac{1}{2}\left[\left(1 - \frac{1}{N}\right)\frac{1}{y} + \gamma y\right] w_y + \frac{1}{2}\left[\left(1 - \frac{1}{N}\right)\frac{1}{y^2} + 2d - \gamma\right] w \ ,$$

$\gamma = d + 2$, which does not preserve the usual Lee–Yang property. However, the viscosity $N \to \infty$ limit equation can be explicitly solved by the method of characteristics and the renormalization group trajectories of the $O(N)$ Heisenberg model be controlled by large deviation. In this talk central limit theorems of $N \to \infty$ PDE model will be presented.

References

[W] Hiroshi Watanabe. "Triviality of Hierarchical $O(N)$ Spin Model in Four Dimensions with Large N" Journ. Stat. Phys. **115**, 1669-1713 (2004).