Jacobian of the Generalised Procrustes Analysis, and Application in Image Segmentation

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The Generalised Procrustes Analysis (GPA) has been extensively used in image registration problems to find the relative displacement of structures that appear in different images. The GPA corresponds to the analytical solution of the problem of finding the affine transformation that minimise the Euclidean distance between two sets with identical number of corresponding points. A few segmentation algorithms have integrated the spatial information provided by the GPA, increasing robustness to occlusion or other image artifacts that produce poor defined boundaries of the objects of interest. However, the resulting algorithms have been designed as heuristic collections of multiple modules, which need a considerably amount of human interaction.

Computing the Jacobian of the GPA it is possible to evaluate, in a variational sense, the shape dissimilarity of two sets of corresponding points. This shape dissimilarity measure can be introduced as a penalising term to those variational image segmentation techniques, such as Snakes or Active Contours, so that the segmentation process favours predefined shapes. Considering a training process to get some information of the shape to be segmented, the integration of a shape dissimilarity term to a Snake increases significantly the robustness to poor defined boundaries. Additionally, this approach transforms a GPA-based segmentation algorithm into the minimisation of a single energy functional, so that the algorithm is considerably more autonomous.

The algorithm has been tested over synthetic and real two and three dimensional Magnetic Resonance Images, obtaining reliable results with subpixel accuracy.