

A CHARACTERISTIC MARTINGALE RELATED TO EXTREMES

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ABSTRACT. Let (X_n) be a sequence of nonnegative, integrable, independent and identically distributed random variables, with common distribution function F . We consider the problem of finding all distribution functions F such that $N_n - cM_n$ is a discrete time martingale, where N_n is the counting process of upper records, $M_n = \max\{X_1, \dots, X_n\}$ is the process of partial maxima and c is a positive constant. We solve the problem by giving explicitly the solutions with finite support and using this for constructing the solutions for the general case by a limiting process. We show that the set of solutions can be parameterized by their support and the mass at the leftmost point of the support.

In contrast to most characterization results, we do not impose restrictions on the class of distribution functions under consideration aside from integrability, necessary to define conditional expectations. As particular cases we consider discrete distributions, with finite or lattice support, and continuous distributions. In the latter two instances we show that, up to a simple transformation, the geometric and the exponential distributions are the unique solutions respectively.

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