MAXIMUM PRINCIPLES, SLIDING TECHNIQUES AND APPLICATIONS TO NONLOCAL EQUATIONS

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ABSTRACT

This paper is devoted to the study of maximum principles holding for some nonlocal diffusion operators defined in (half-) bounded domains and its applications to obtain qualitative behaviors of solutions of some nonlinear problems. I show that, as in the classical case, the nonlocal diffusion considered satisfies a weak and a strong maximum principle. Uniqueness and monotonicity of solutions of nonlinear equations are therefore expected as in the classical case. I first present a simple proof of this qualitative behavior and the weak/strong maximum principle. An optimal condition to have a strong maximum for operator $M[u] := J * u - u$ is also obtained. The proofs of the uniqueness and monotonicity essentially relies on the sliding method and the strong maximum principle.

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