

# The internal structure of combustion waves: singular perturbation analysis

Grigori Chapiro<sup>1</sup>, Alexei Mailybaev<sup>2</sup>, Aparecido J. de Souza<sup>3</sup>, Dan Marchesin<sup>4</sup>

## Abstract

In-situ combustion is a technique with potential for recovery in heavy oil reservoirs. The technique is to inject air and to utilize heavy and immobile components of the crude oil as a fuel to produce in-place heat necessary for decreasing the oil viscosity. Thus, increase in temperature near the propagating combustion front and the transport of the produced thermal energy have a big impact on the recovery.

The flow of air and oil in a petroleum reservoir is governed by a system of conservation laws - equations for mass, energy, momentum and enthalpy. We examine solutions of these partial differential equations in the form of shock waves that are limits of traveling waves of the parabolic system. These solutions should satisfy the system of ordinary differential equations associated with partial differential equations and “join” left and right shock states. Furthermore, we show how heteroclinic orbits appear that “join” both equilibrium points defined by these states.

We also discuss a method for solving this kind of ordinary differential equations by writing the fields associated to our differential equation system in asymptotic series and use the singular perturbation method to determine the solution up to first order. We show reasonable conditions for the existence of traveling waves in the asymptotic series context.

We use these results to determine when the combustion wave in a porous media exists or when it extinguishes.

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<sup>1</sup>Instituto Nacional de Matematica Pura e Aplicada - IMPA, Estrada Dona Castorina, 110, 22460-320 Rio de Janeiro RJ, Brazil. E-mail: grigori@fluidimpa.br.

<sup>2</sup>Institute of Mechanics, Moscow State University, Michurinsky pr. 1, 119192 Moscow, Russia. E-mail: mailybaev@imec.msu.ru.

<sup>3</sup>Dep. de Matematica e Estatistica, Universidade Federal de Campina Grande - UFCG, 58109-907 Campina Grande PB, Brazil. E-mail: cido@dme.ufpb.br

<sup>4</sup>Instituto Nacional de Matematica Pura e Aplicada - IMPA, Estrada Dona Castorina, 110, 22460-320 Rio de Janeiro RJ, Brazil. E-mail: marchesin@fluidimpa.br.