

Stable stationary solutions of the Vlasov equation as metastable states of particle systems in the mean-field limit

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Abstract

A system of N particles interacting via a smooth two-body potential, in the mean field limit, is described by the Vlasov equation. The convergence of the particle system to the Vlasov equation is proved for times as long as $\log N$.

What we can say for longer times?

We are interested here in the case in which initially the particles are independently extracted from a stable stationary solution of the Vlasov equation. In this case it seems reasonable that the particle system remains close to the initial density profile for times much longer than $\log N$. In other words this means that the stable stationary solution of the Vlasov equation can be seen as metastable states for the particle system.

Here I will briefly discuss some the recent physical literature on the argument and then I will give some rigorous results about this problem obtained in a collaboration with Frederic Rousset.

In particular we prove that when the particles are initially extracted from a stationary stable distribution of the Vlasov equation then they remain close to it for times as long as N^α for some $\alpha > 0$.