On well-posedness of balance equations for atmosphere dynamics

Andrei Bourchtein, Ludmila Bourchtein
Institute of Physics and Mathematics
Pelotas State University, Brazil
E-mail: burstein@terra.com.br

The partial differential equations of atmosphere dynamics represent a complex system with multiple space and time scales. Even commonly used filtered equations, such as hydrostatic and shallow water ones, continue to be stiff systems, which contain fast gravitational waves as well as relatively slow synoptic processes. The time spectrum of actual large scale motions is very selective and contains the atmospheric waves with characteristic period of one-two days, while faster waves have quite small amplitudes. It is natural to expect a similar behavior from the solutions of mathematical models. However, it is well-known that solution form depends strongly on the type of initial values and dynamically inconsistent set of initial conditions, though mathematically correct, can generate the fast gravity waves of great amplitude, which are not observed in the real atmosphere.

Modern weather forecasting and modeling consists of numerical solution of the governing partial differential equations. The initial conditions for numerical weather prediction are supplied by data assimilation schemes and they are usually not adjusted dynamically. Therefore some corrections to initial values have to be done to avoid physically unrealizable oscillations. In this study we consider the well-known and widespread approaches to initial adjustment: the nonlinear normal mode initialization and the bounded derivative method. In both cases the initial balance conditions of atmospheric fields are expressed in the form of time-independent partial differential systems to be solved with appropriate boundary conditions. Using concept of ellipticity in the sense of Douglis-Nirenberg, we derive the conditions of well-posedness of boundary value problems for balance equations subject to different physical constraints: prescribed pressure field, given vorticity or potential vorticity field. We also consider some examples of the real atmosphere circulations when the obtained conditions are violated.