Calculus of variations under uniform constraints on the gradient: existence and approximation of Lagrange multipliers via p-Laplacian like penalty

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Abstract. Many mathematical models in physics and engineering sciences consist in solving variational problems where the criteria to be minimized depend on scalar or vector potentials whose gradients are subject to uniform constraints. This is the case, for instance, in the optimal design of mechanical structures where maximal stresses must satisfy some pointwise bounds to avoid material failure, or in the optimal control of thermodynamical systems in which point concentrations of high magnitude heat flow are not desirable. In this talk, we will discuss some recent theoretical and numerical results for this type of uniformly constrained variational problems. The approach we will follow is based on the introduction of a p-power integral term into the objective functional so that constraint violation is penalized. In terms of the correspondig Euler-Lagrange system, the penalty functional is associated with a p-Laplacian like operator. The goal is to understand the asymptotic behavior of the resulting unconstrained problem as the penalty parameter p tends to infinity. We will show that the path of optimal solutions that is traced by continuously varying p is compact for an appropriate functional topology, and its cluster points solve the original problem. In so doing, we will identify an explicit dual path parametrized by p, and we will give conditions ensuring that its cluster points are Lagrange multipliers for the constraints. Such limit multipliers satisfy, in a weak sense, a generalized primal/dual differential system. The theoretical results will be illustrated through some numerical experiments in the scalar case.