Transitional Waves in Three-Phase Flows in Heterogeneous Petroleum Reservoirs

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1 Introduction

Natural reservoirs are porous and permeable rocks containing deposits of hydrocarbons (oil reservoirs) or pure water (aquifers). Porosity and permeability are the most important physical properties of reservoirs that affect flow and transport. In general these geological parameters are stochastically modeled (see [**?**, **?**] and references therein). As a consequence the equations governing flow and transport in porous media are stochastic and predictions based on them (e.g. hydrocarbon production forecasts in petroleum reservoirs or prediction of contaminant movement in groundwater) are uncertain. In order to assess the influence of geological uncertainty in fluid flow in porous media, stochastic numerical simulators are commonly used. In this manuscript we are concerned with the numerical simulation of three-phase flow (water, gas and oil) in petroleum reservoirs.

In immiscible three-phase oil, gas and water flows, typical of petroleum reservoirs, the leading oil front can split into two, a classical Buckley-Leverett front followed by a new type of shock wave [**?**]. This new shock wave is related to the existence of an elliptic region or an umbilic point for the system of nonlinear conservation laws describing the convective transport of the fluid phases. Unlike classical shock waves (e.g., Buckley-Leverett fronts), this non-classical "transitional" shock wave is very sensitive to the form of the parabolic terms in the equations that arise from diffusive effects. It is then imperative that capillary pressure effects be modeled accurately in order to calculate the physically correct transitional waves [**?**, **?**].

We describe a numerical procedure, based on a two-level operator splitting technique, for three-phase flows that takes into account capillary pressure (dispersive forces) [**?**, **?**, **?**, **?**]. This numerical procedure has been used to indicate the existence of non-classical transitional waves in multidimensional heterogeneous flows [**?**, **?**], thereby ex-

tending previous results for one-dimensional problems [**?**, **?**]. The authors are currently investigating, with the numerical procedure developed, the stability (with respect to viscous fingering) of transitional waves in heterogeneous formations as a first step in the investigation of the scale-up problem for three-phase flows.

Our numerical procedure combines a non-oscillatory, second order, conservative central difference scheme [**?**] for the system of conservation laws modeling the convective transport of the fluid phases with locally conservative mixed finite elements for the associated parabolic and elliptic problems [**?**] (see also [**?**, **?**, **?**]).

Preliminary results concerning the investigation of interaction of variable porosity and permeability and transitional waves on three-phase flows in two-dimensional space dimension will be also presented.

2 Governing Equations of the Three-Phase Flows

We consider two-dimensional, horizontal flow of three immiscible, incompressible fluid phases in a porous medium. For concreteness, the phases will be called gas, oil, and water and indicated by the subscripts g, o, and w, respectively. We assume that there are no internal sources or sinks. Compressibility, mass transfer between phases, and thermal effects are neglected.

We assume that the three fluid phases saturate the pores; thus, with S_i denoting the saturation (local volume fraction) of phase i, $\sum_i S_i = 1$, $i = g, o, w$. Consequently, any pair of saturations inside the triangle of saturations $\triangle := \{\, (S_i, S_j) \,:\, S_i, S_j \geq 0,\, S_i + S_j \leq 1,\, i \neq j\}$ can be chosen to describe the state of the fluid. Therefore, the equations governing the three-phase flow are as follows:

Saturation equations:

$$\frac{\partial}{\partial t}(\phi(\mathbf{x})S_w) + \nabla \cdot (\mathbf{v} f_w(S_w, S_g)) = \nabla \cdot \mathbf{w}_w \tag{1}$$

$$\frac{\partial}{\partial t}(\phi(\mathbf{x})S_g) + \nabla \cdot (\mathbf{v} f_g(S_w, S_g)) = \nabla \cdot \mathbf{w}_g, \tag{2}$$

In our model we shall work with the saturations S_w and S_g of water and gas, respectively. The (diffusive) flux terms $\mathbf{w}_w$ and $\mathbf{w}_g$ are given by

$$[\mathbf{w}_w, \mathbf{w}_g]^T = K(\mathbf{x})\, B(S_w, S_g)\, [\nabla S_w, \nabla S_g]^T . \tag{3}$$

Here, $[\mathbf{a}, \mathbf{b}]$ denotes the 2-by-2 matrix with column vectors $\mathbf{a}$ and $\mathbf{b}$, and $B = QP'$, where

$$Q(S_w, S_g) = \begin{bmatrix} \lambda_w(1 - f_w) & -\lambda_w f_g \\ -\lambda_g f_w & \lambda_g(1 - f_g) \end{bmatrix}, \quad P'(S_w, S_g) = \begin{bmatrix} \dfrac{\partial p_{wo}}{\partial S_w} & \dfrac{\partial p_{wo}}{\partial S_g} \\ \dfrac{\partial p_{go}}{\partial S_w} & \dfrac{\partial p_{go}}{\partial S_g} \end{bmatrix}. \tag{4}$$

In the above, $K(\mathbf{x})$ and $\phi(\mathbf{x})$ are the absolute permeability and the rock porosity of the porous medium, respectively. $\lambda_i(S_w, S_g) = k_i/\mu_i$ denote the phase mobilities, given in terms of the phase relative permeabilities k_i and phase viscosities μ_i. $f_i(S_w, S_g) = \lambda_i/\lambda$ is the fractional flow function of phase i. The diffusive (capillary) term is represented by the right-hand side of the system (**??**)-(**??**). For our model one can verify that this term is strictly parabolic in the interior of the saturation triangle $\triangle$ [**?**, **?**].

Pressure-Velocity equations:

$$\nabla \cdot \mathbf{v} = 0, \tag{5}$$

$$\mathbf{v} = -K(\mathbf{x})\lambda(S_w, S_g)\nabla p_o + \mathbf{v}_{wo} + \mathbf{v}_{go}, \tag{6}$$

where $\mathbf{v}_{wo}$ and $\mathbf{v}_{go}$ are the correction velocities, defined by

$$\mathbf{v}_{ij} = -K(\mathbf{x})\lambda_i(S_w, S_g)\nabla p_{ij}, \tag{7}$$

and $p_{ij} = p_i - p_j$ denote the capillary pressure between phases i and j, $i \neq j$, which are experimentally measured as functions of the saturations; p_o is the oil pressure. Also, $\lambda_i(S_w, S_g) = k_i/\mu_i$ denote the phase mobilities, given in terms of the phase relative permeabilities.

We refer the reader to [**?**, **?**] for a detailed description of the derivation of the phase formulation of the governing equations of three-phase flows. Boundary and initial conditions for the full coupled (**??**)-(**??**) must be imposed to complete the definition of the mathematical model; in particular, S_w and S_g must be specified at the initial time $t = 0$. The boundary conditions will be introduced in the description of the operator splitting for the numerical method.

Different approaches for solving numerically the three-phase flow equations can be found in [**?**, **?**, **?**].

3 The Numerical Simulator

We employ a two-level operator-splitting procedure (see [**?**]) for the numerical solution of the three-phase flow system (**??**)-(**??**). The splitting allows time steps for the pressure-velocity calculation that are longer than those for the diffusive calculation, which are in turn longer than those for advection. Thus, we introduce three time steps: Δt_c for the solution of the hyperbolic problem for the advection, Δt_d for the diffusive calculation and Δt_p for the pressure-velocity calculation so that $\Delta t_p \geq \Delta t_d \geq \Delta t_c$. We remark that in practice, variable time steps are always useful, especially for the advection micro-steps subject dynamically to a CFL condition.

The oil pressure and the Darcy velocity are approximated at times $t^m = m\Delta t_p$, $m = 0, 1, 2, \ldots$. Locally conservative mixed finite elements are used to discretize the pertinent elliptic equation (see [**?**, **?**]).

The saturations S_w and S_g are approximated at times $t_n = n\Delta t_d$, $n = 1, 2, \cdots$ in the diffusive calculation; recall that they are specified at $t = 0$. Locally conservative mixed

finite elements are used to discretize the spatial operators in the diffusion system; the time discretization of the latter is performed by means of the implicit backward Euler method (see [**?**]).

In addition, there are values for the saturations computed at intermediate times $t_{n,\kappa} = t_n + k\Delta t_c$ for $t_n < t_{n,\kappa} \leq t_{n+1}$ that take into account the advective transport of water and gas but ignore the diffusive effects. In these intermediate times the system of conservation laws is approximated by a non-oscillatory, second order, conservative central difference scheme (see [**?**, **?**]).

We indicate the references [**?**, **?**] to the readers for a detail description of the fractional-step procedure.

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